Have your pi and e it too.

By Derek Buchanan

It has recently been asserted by someone at UNSW that $\pi^4 + \pi^5 = e^6$.
This is not true and actually I'd be more inclined to present a

Proposition. $\pi^4 + \pi^5 \neq e^6$

and several proofs follow.

Proof 1. By decimal expansion, $\pi^4 + \pi^5 = 403.4287\mathbf{7}... < 403.4287\mathbf{9}... = e^6$ □

Proof 2. Using rational numbers only,

$\pi < 3.14159266$ & $2.718281828 < e$ and so

$$\begin{aligned} \pi^4 + \pi^5 &< 3.14159266^4 + 3.14159266^5 \\ &= 403.428779736372553284665758501673375658857 6 \\ &< 403.428793083965001476126676589903866851868865301397704704 \\ &= 2.718281828^6 \\ &< e^6 \end{aligned}$$ □

Proof 3. Using integers only,

$e \times 10^9 > 2718281828$ and $\pi \times 10^8 < 314159266$ and so

$$\begin{aligned} (e^6 - \pi^4 - \pi^5) \times 10^{54} &> 2718281828^6 - 314159266^4 \times 10^{22} - 314159266^5 \times 10^{14} \\ &= 1334759244819146091808823049119301126530139770470 4 \\ &> 0 \end{aligned}$$

Hence $\pi^4 + \pi^5 \neq e^6$ □

Proof 4. By algebra,

$\pi^x + \pi^{x+1} = e^{x+2} \Rightarrow x = \frac{2 - \ln(\pi+1)}{\ln \pi - 1} = 4.0000003027... \neq 4$ and so $\pi^4 + \pi^5 \neq e^6$ □

Proof 5. By continued fractions,

$$\begin{aligned}
\pi^4 + \pi^5 &= 403 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{\mathbf{99} + \cfrac{1}{\vdots}}}} \\
&\neq 403 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{\mathbf{91} + \cfrac{1}{\vdots}}}} \\
&= e^6
\end{aligned}$$

□