

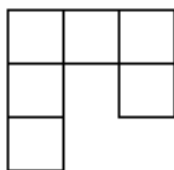
International Mathematical Olympiad, 2004, Athens, Greece

1. Let ABC be an acute-angled triangle with $AB \neq AC$. The circle with diameter BC intersects the sides AB and AC at M and N respectively. Denote by O the midpoint of the side BC . The bisectors of the angles $\angle BAC$ and $\angle MON$ intersect at R . Prove that the circumcircles of the triangles BMR and CNR have a common point lying on the side BC .

2. Find all polynomials f with real coefficients such that for all reals a, b, c such that $ab + bc + ca = 0$ we have the following relation:

$$f(a - b) + f(b - c) + f(c - a) = 2f(a + b + c).$$

3. Define a “hook” to be a figure made up of six unit squares as shown below in the picture, or any of the figures obtained by applying rotations and reflections to this figure.



Determine all $m \times n$ rectangles that can be covered without gaps and without overlaps with hooks such that

- the rectangle is covered without gaps and without overlaps
- no part of a hook covers area outside the rectangle

4. Let $n \geq 3$ be an integer. Let t_1, t_2, \dots, t_n be positive real numbers such that

$$n^2 + 1 > (t_1 + t_2 + \dots + t_n) \left(\frac{1}{t_1} + \frac{1}{t_2} + \dots + \frac{1}{t_n} \right).$$

Show that t_i, t_j, t_k are side lengths of a triangle for all i, j, k with $1 \leq i < j < k \leq n$.

5. In a convex quadrilateral $ABCD$ the diagonal BD bisects neither the angle ABC nor the angle CDA . A point P lies inside $ABCD$ and satisfies

$$\angle PBC = \angle DBA \text{ and } \angle PDC = \angle BDA$$

Prove that $ABCD$ is a cyclic quadrilateral if and only if $AP = CP$.

6. We call a positive integer *alternating* if every two consecutive digits in its decimal representation are of different parity.

Find all positive integers n such that n has a multiple which is alternating.