Mathematics Extension 2

Objective–response questions

Marks

Multiple choice

1 An object of mass m is projected downwards from point P with an initial velocity of 1 ms^{-1}. The object then falls under the influence of gravity in a medium which provides resistance proportional to velocity. Take the initial position as $y = 0$ and downwards as the positive direction. **1**

Which of the following initial value problems most accurately reflects this situation? (The acceleration due to gravity is g, and k is a positive constant).

(A) $\ddot{y} + k\dot{y} = mg; \quad y(0) = 0, \quad \dot{y}(0) = 1$

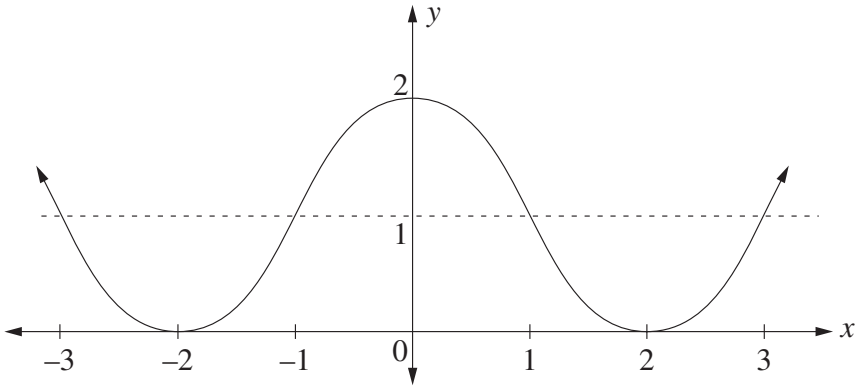
(B) $\ddot{y} + k\dot{y} = g; \quad y(0) = 0, \quad \dot{y}(0) = 1$

(C) $\ddot{y} - k\dot{y} = mg; \quad y(0) = 0, \quad \dot{y}(0) = 1$

(D) $\ddot{y} + k\dot{y} = g; \quad y(0) = 0, \quad \dot{y}(0) = -1$

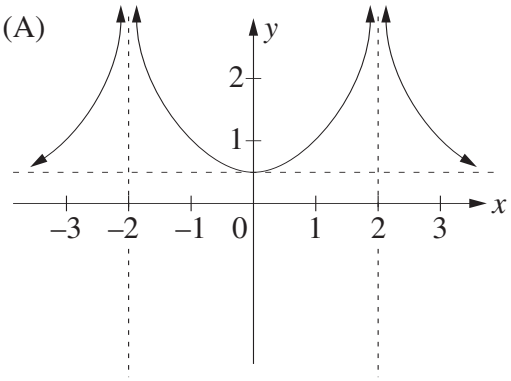
Marks

2 The graph of the function $y = f(x)$ is shown below. **1**

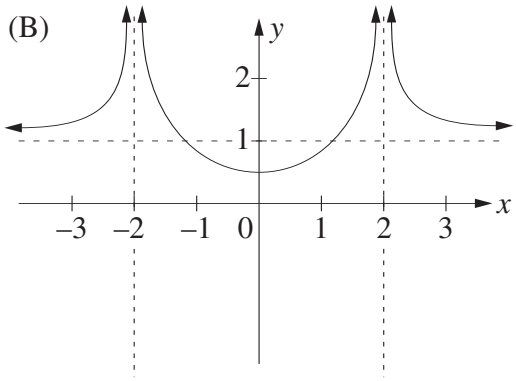


Which graph shows the most accurate representation of the graph of $y = \dfrac{1}{f(x)}$?

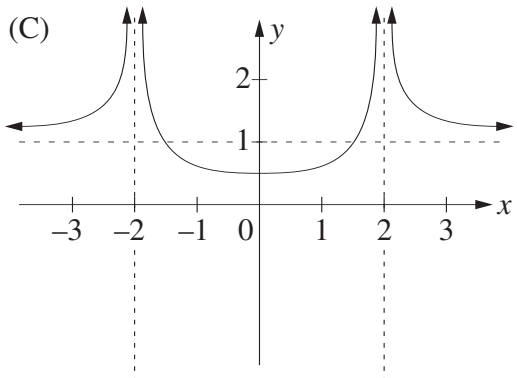
(A)



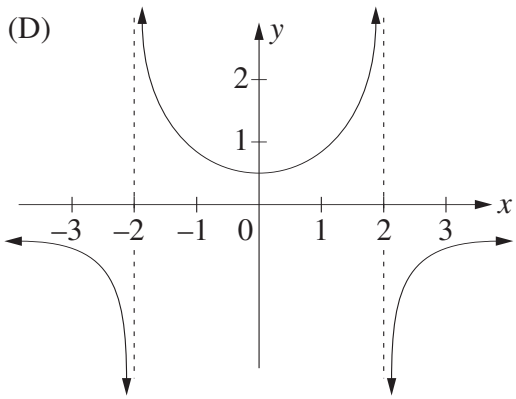
(B)



(C)

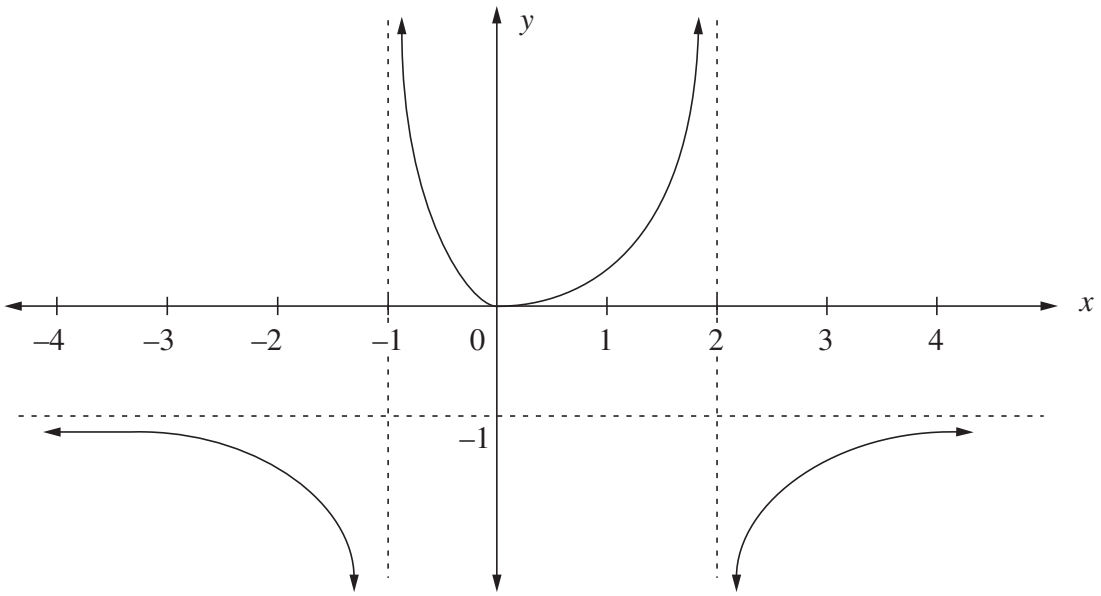


(D)



Marks

3 The graph of the function $y = \dfrac{g(x)}{(x-2)(x+1)}$ is shown below. **1**



The function $g(x)$ could be

(A) x.

(B) $-x$.

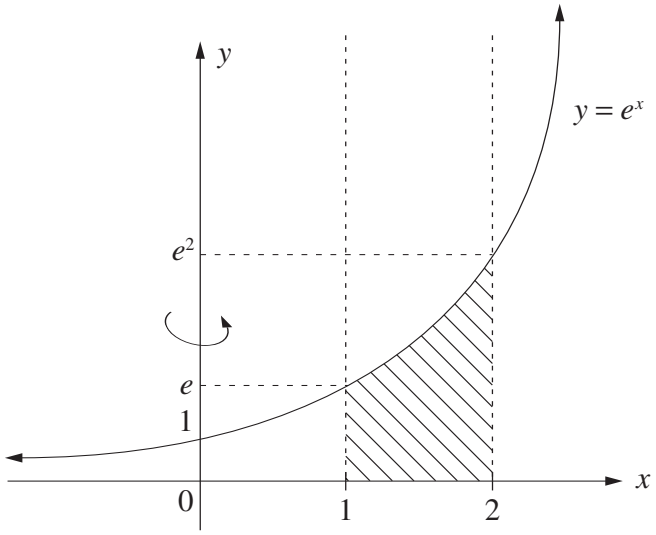
(C) x^2.

(D) $-x^2$.

Marks

Multiple correct/incorrect

4 The region bounded by the curve $y = e^x$, the x-axis, and the lines $x = 1$ and $x = 2$ is rotated around the y-axis to form a solid with volume V. **2**



Indicate whether each of the following is Correct or Incorrect.

a. $V = \pi \int_e^{e^2} \left[4 - (\ln y)^2\right] dy$

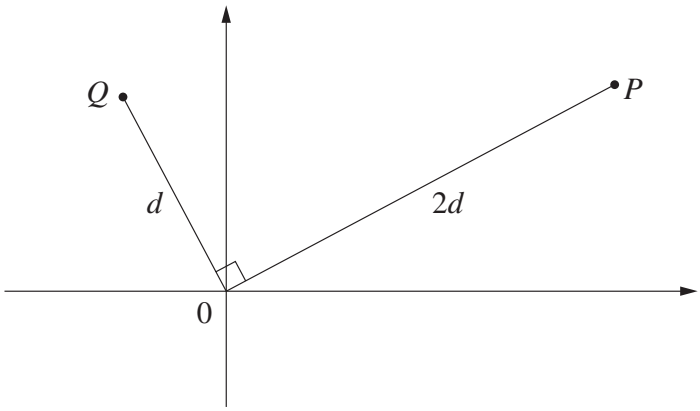
b. $V = 3\pi e + \pi \int_e^{e^2} \left[4 - (\ln y)^2\right] dy$

c. $V = 3\pi e + \pi \int_e^{e^2} (4 - 2\ln y)\, dy$

d. $V = 2\pi \int_1^2 xe^x dx$

Marks

Use the information below to answer Questions 5 and 6.



NOT TO SCALE

In the Argand diagram, the points P and Q represent the complex numbers z and w, respectively. Distance OP is $2d$ units, and distance OQ is d units. Angle QOP is a right angle.

5 Indicate whether each of the following is Correct or Incorrect. **2**

a. $w = \dfrac{iz}{2}$

b. $w = -\dfrac{iz}{2}$

c. $z = i\overline{w}$

d. $w = -\dfrac{z}{2i}$

6 Indicate whether each of the following is Correct or Incorrect. Assume that all arguments are given as principal values. **2**

a. $\arg(z) + \arg(\overline{z}) = 0$

b. $\arg(w) - \arg(z) = \dfrac{\pi}{2}$

c. $\arg(z) - \arg(w) = \dfrac{\pi}{2}$

d. $\arg(z) + \arg(\overline{w}) = -\dfrac{\pi}{2}$

Marks

Constrained response

7 Let $z = 4 + i$ and $w = \overline{z}$. Find each of the following in the form $x + iy$.

(a) w **1**

(b) z^2 **1**

Marks

Free-response questions

8 A mass-spring system is modelled by the second order differential equation **4**

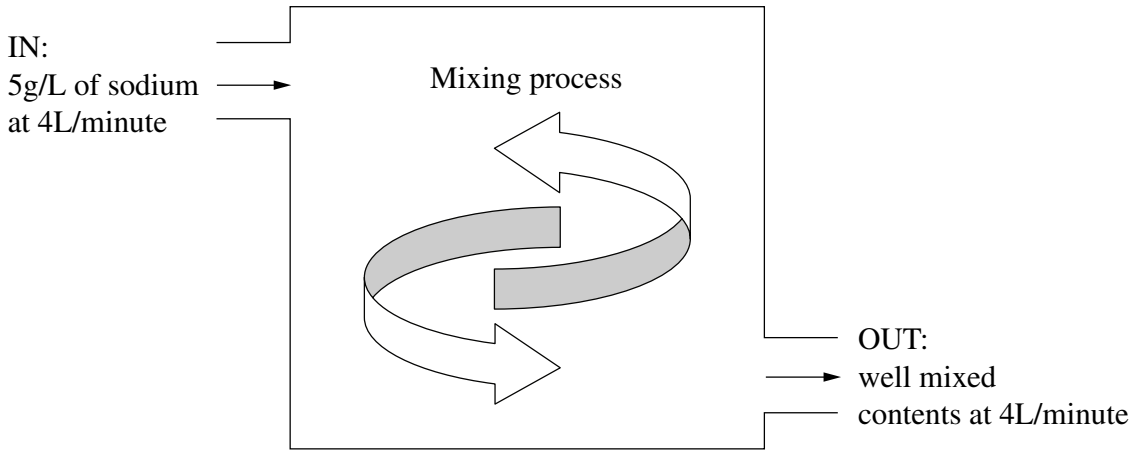
$$2\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 5x = 0$$

where $x = x(t)$ gives the displacement of the mass at time t seconds.

Solve this differential equation, given the initial conditions are given by $x(0) = -1$ and $\dot{x}(0) = 0$.

Marks

9 In a mixing process, a solution containing 5 grams of sodium per litre is pumped into a mixing tank at a rate of 4 litres per minute. The fluid in the tank is mixed before being pumped out at a rate of 4 litres per minute. The tank initially contains 240 litres of fresh water.



Let the amount of sodium in the tank at time t minutes be A grams.

(a) By considering the rate of change of sodium in the tank as the difference between the rates of sodium input and output, show that the differential equation for the amount of sodium in the tank at time t minutes is **2**

$$\frac{dA}{dt} = \frac{1200 - A}{60}.$$

(b) Solve the differential equation given in part (a), given that there is initially no sodium in the tank. **3**

(c) Find the concentration of sodium in the tank after 3 hours. Give your answer to 2 decimal places. **1**

(d) Find $\lim_{t\to\infty} A(t)$ and explain the meaning of your answer in the context of the problem. **2**

(e) Make a neat sketch of $A(t)$ versus t. **2**

(f) Find the maximum value of $\frac{dA}{dt}$ and explain the meaning of your answer in the context of the problem. **2**